Accelerating Diffusion Sampling with Optimized Time Steps

Shuchen Xue¹ · Zhaoqiang Liu² · Fei Chen³ · Shifeng Zhang³ · Tianyang Hu³ · Enze Xie³ · Zhenguo Li³ 1AMSS, CAS 2UESTC 3Huawei Noah's Ark Lab



Diffusion Models



- Getting noise from data is easy (Forward SDE).
- Generating data by reversing the forward process.

Image from Song et al., 2020

Estimating the score function by Denoising Score matching (Vincent 2010).

$$\boldsymbol{\theta}^* = \operatorname*{arg\,min}_{\boldsymbol{\theta}} \mathbb{E}_t \Big\{ \lambda(t) \mathbb{E}_{\boldsymbol{x}_0} \mathbb{E}_{\boldsymbol{x}_t \mid \boldsymbol{x}_0} \big[\big\| \boldsymbol{s}_{\boldsymbol{\theta}}(\boldsymbol{x}, t) - \nabla_{\boldsymbol{x}_t} \log p_{0t}(\boldsymbol{x}_t \mid \boldsymbol{x}_0) \big\|_2^2 \big] \Big\}$$

Accelerating Diffusion Sampling with Optimized Time Steps

Target: Given pretrained diffusion model, budget of N network inferences and a prescribed diffusion sampler, what is the optimal discretization scheme of time steps?

Method: Derive a theory-based surrogate optimization objective which can be solved utilizing existing optimization method (e.g. constrained trust region method)

Advantage: Negligible computational cost lightweight optimization can be solved within tens of seconds. Optimization objective: minimize which is the distance between ground truth solution and the numerical solution (related to selected timesteps)

Assumption: the score matching error are bounded in L2 norm

Assumption 1. For any $t \in \{t_0, t_1, \ldots, t_N\}$, the error in the score estimate is bounded in $L^2(q_t)$:

 $\begin{aligned} \|\nabla_{\mathbf{x}} \log q_t - \mathbf{s}_{\boldsymbol{\theta}}(\cdot, t)\|_{L^2(q_t)}^2 \\ &= \mathbb{E}_{q_t} \left[\|\nabla_{\mathbf{x}} \log q_t(\mathbf{x}) - \mathbf{s}_{\boldsymbol{\theta}}(\mathbf{x}, t)\|^2 \right] \le \eta^2 \varepsilon_t^2, \end{aligned}$

where $\eta > 0$ is an absolute constant.

Accelerating Diffusion Sampling with Optimized Time Steps

Lemma 1. For any $\mathbf{x}_0 \sim q_0$ and $P_0 \in (0, 1)$, with probability at least $1 - P_0$, the following event occurs: For all $t \in \{t_0, t_1, \ldots, t_N\}$ and $\mathbf{x}_t \sim q_t$, we have

$$\|\mathbf{x}_{\boldsymbol{\theta}}(\mathbf{x}_t, t) - \mathbf{x}_0\| \le \tilde{\eta}\tilde{\varepsilon}_t$$

where
$$\tilde{\eta} := \sqrt{\frac{N+1}{P_0}} \eta$$
 and $\tilde{\varepsilon}_t := \frac{\varepsilon_t \sigma_t^2}{\alpha_t}$.

$$\tilde{\mathbf{x}}_{\epsilon} = \frac{\sigma_{\epsilon}}{\sigma_{T}} \mathbf{x}_{T} + \sigma_{\epsilon} \sum_{n=1}^{N} \sum_{j=0}^{k_{n}-1} w_{n;k_{n},j} \boldsymbol{f}(\lambda_{n-k_{n}+j}) \\ \|\tilde{\mathbf{x}}_{\epsilon} - \mathbf{x}_{0}\|$$

$$\min_{\lambda_{t_1},\dots,\lambda_{t_{N-1}}} \sum_{i=0}^{N-1} \tilde{\varepsilon}_{t_i} \cdot \left| \sum_{n-k_n+j=i} w_{n;k_n,j} \right|$$
s.t. $\lambda_{t_{n+1}} > \lambda_{t_n}$, for $n = 0, 1, \dots, N-1$,

Solve the problem use constrained trust region method

Algorithm 1 Finding the time steps via (35)

- **Require:** Number of time steps N, initial time of sampling T, end time of sampling ϵ , any sampling algorithm that is characterized by local polynomials $\mathcal{P}_{n;k_n-1}(\lambda)$ of the form (10), the function $\tilde{\varepsilon}_t := \frac{\sigma_t^p}{\alpha_t}$ with a fixed positive integer p for score approximation
- 1: Set $\lambda_{t_0} = \lambda_T$ and $\lambda_{t_N} = \lambda_{\epsilon}$ and calculate $\tilde{\varepsilon}_{t_i}$ for $i = 0, 1, \dots, N-1$
- 2: Calculate the weights $w_{n;k_n,j}$ from (18).
- 3: Solve the optimization problem in (35) via the constrained trust region method.
- 4: **Return**: Optimized λ (or equivalently, time) steps $\hat{\lambda}_{t_1}, \ldots, \hat{\lambda}_{t_{N-1}}$.

Results on benchmarks



Visualization



Visualization on ImageNet 256 DIT-XL-2 model

Results on t2i



Text-to-Image generation results on PixArt- α

Results on t2i

NFE = 5

A alpaca made of colorful building blocks, cyberpunk



Uniform-l

Optimized Steps (Ours)

bird's eye view of a city



NFE = 5



Text-to-Image generation results on PixArt- α

Running time and Summary

seconds	on CPU
---------	--------

NFEs	5	6	7	8	9	10	12	15
Time(s)	1.9	2.3	5.3	5.9	7.8	8.8	11.0	14.1

Table 2. Running time of our optimization algorithm.

- We propose an optimization-based method to find appropriate time steps to accelerate the sampling of diffusion models.
- Experimental results on popular image datasets demonstrate that our method can be employed in a plug-and-play manner and achieves state-of-the-art sampling performance based on various pretrained diffusion models
- Code is available at https://github.com/scxue/DM-NonUniform