Scalable 3D Registration via Truncated Entry-wise Absolute Residuals

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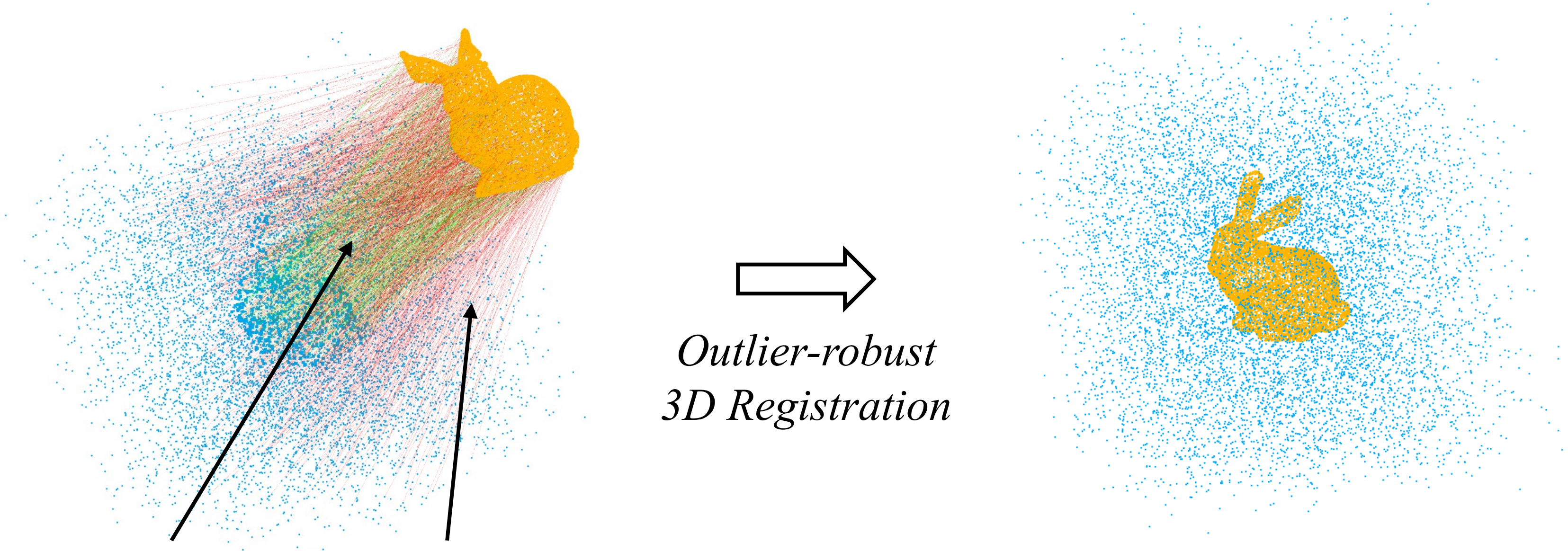
Background: Outlier-robust 3D Registration



Outlier-contaminated
Correspondences

Inliers

Outliers

Outlier-robust
3D Registration

Existing Problem: Scalability & Efficiency

Method	**Robustness**	**Efficiency**	**Scalability**
Alternating Minimization	×	√	√
RANSAC (Few Iterations)	×	√	√
RANSAC (Huge Iterations)	√	×	√
Semidefinite Programs	√	×	×
Consistency Graph-based	√	√/×	×
Outlier Removal	√	×	√
Deep Learning-based	√/×	×	×

Our Contribution: Truncated Entry-wise Absolute Residuals

- Classical Robust Losses

$$\max_{(\boldsymbol{R},\boldsymbol{t})\in\mathrm{SE}(3)} \sum_{i=1}^{N} \mathbf{1}\left(\|\boldsymbol{y}_i - \boldsymbol{R}\boldsymbol{x}_i - \boldsymbol{t}\|_2 \le \xi_i\right), \qquad \text{(CM)}$$

$$\min_{(\boldsymbol{R},\boldsymbol{t})\in\mathrm{SE}(3)} \sum_{i=1}^{N} \min\left\{\|\boldsymbol{y}_i - \boldsymbol{R}\boldsymbol{x}_i - \boldsymbol{t}\|_2^2,\ \xi_i^2\right\}. \qquad \text{(TLS)}$$

- Our Truncated Entry-wise Absolute Residuals

$$\min_{(\boldsymbol{R},\boldsymbol{t})\in\mathrm{SE}(3)} \sum_{i=1}^{N} \min\left\{\|\boldsymbol{y}_i - \boldsymbol{R}\boldsymbol{x}_i - \boldsymbol{t}\|_1,\ \xi_i\right\}. \qquad \text{(TEAR)}$$

Our Contribution: DoF Decomposition of TEAR

$$\min_{(\mathbf{R},\mathbf{t})\in\mathrm{SE}(3)} \sum_{i=1}^{N} \min\{\|\mathbf{y}_i - \mathbf{R}\mathbf{x}_i - \mathbf{t}\|_1,\ \xi_i\}. \qquad \text{(TEAR)}$$

Decomposition

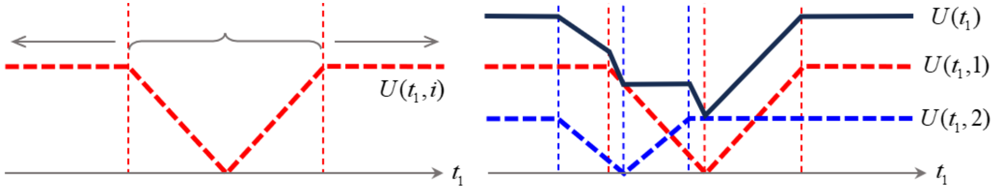
$$\begin{cases} \displaystyle\min_{\mathbf{r}_1\in\mathbb{S}^2, t_1\in\mathbb{R}} \sum_{i=1}^{N} \min\big\{|y_{i1} - \mathbf{r}_1^\top \mathbf{x}_i - t_1|,\ \xi_{i1}\big\}. & \text{(TEAR-1)} \\ \displaystyle\min_{\mathbf{r}_2\in\mathbb{S}^2, t_2\in\mathbb{R}} \sum_{i\in\hat{\mathcal{I}}_1} \min\big\{|y_{i2} - \mathbf{r}_2^\top \mathbf{x}_i - t_2|,\ \xi_{i2}\big\} & \\ \text{s.t.} \qquad \mathbf{r}_2^\top \hat{\mathbf{r}}_1 = 0 & \text{(TEAR-2)} \end{cases}$$

Our Contribution: Tight Bounding Functions

Bounds of (TEAR-1) in a given branch of $\mathbf{r}_1$:

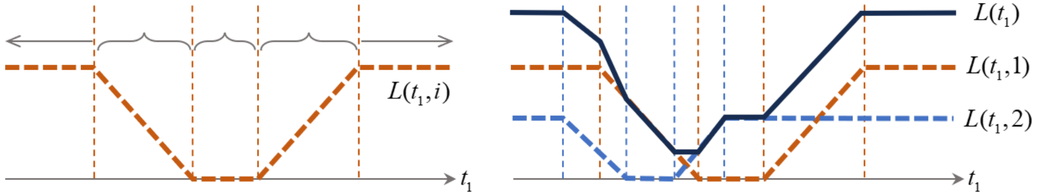
(*Upper Bound*) We choose the center $\dot{\boldsymbol{r}}_1$ and let $a_i := y_{i1} - \dot{\boldsymbol{r}}_1^\top \boldsymbol{x}_i$, then an upper bound can be computed as

$$\overline{U} = \min_{t_1 \in \mathbb{R}} \sum_{i=1}^{N} \min\{|a_i - t_1|,\ \xi_{i1}\}$$
$$= \min_{t_1 \in \mathbb{R}} U(t_1),$$

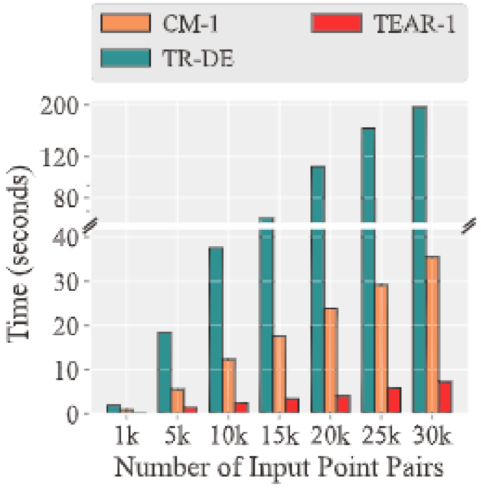


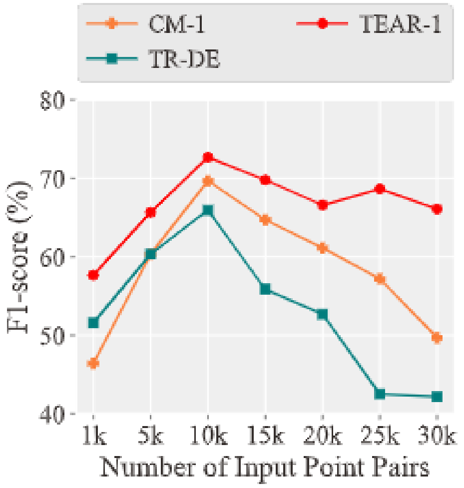
(*Lower Bound*) Let $b_i := y_{i1} - \boldsymbol{r}_1^\top \boldsymbol{x}_i$ and we can compute $b_i \in [b_{il}, b_{iu}]$, then an lower bound can be computed as

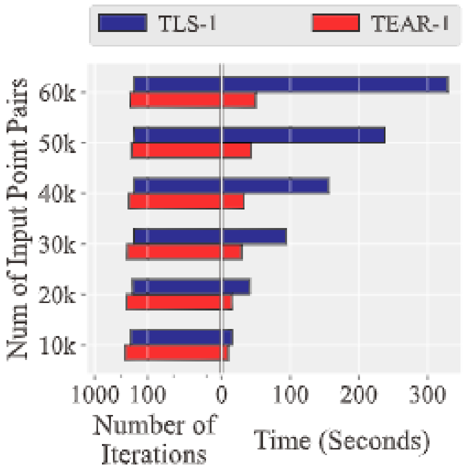
$$\underline{L} = \min_{t_1 \in \mathbb{R},\ b_i \in [b_{il}, b_{iu}]} \sum_{i=1}^{N} \min\{|b_i - t_1|,\ \xi_{i1}\}$$
$$= \min_{t_1 \in \mathbb{R},\ b_i \in [b_{il}, b_{iu}]} \sum_{i=1}^{N} L(t_1, b_i, i),$$

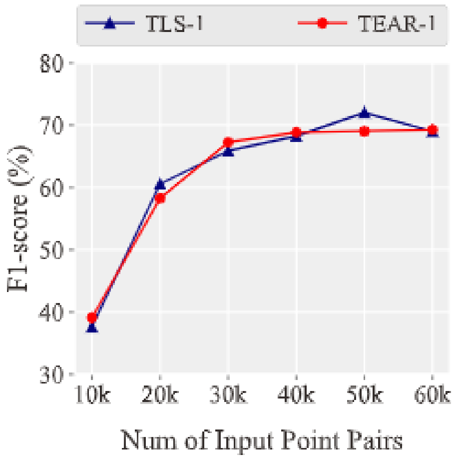


Experiments: TEAR Versus CM and TLS

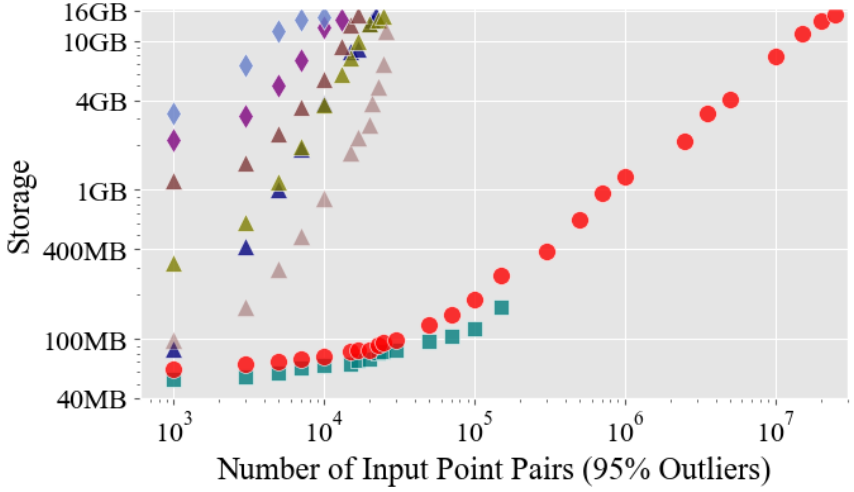


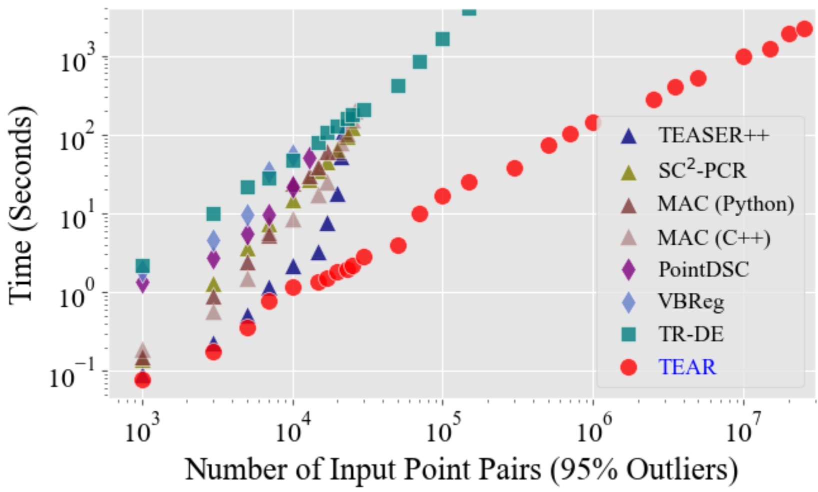






Evaluation Experiments on Scalability and Efficiency

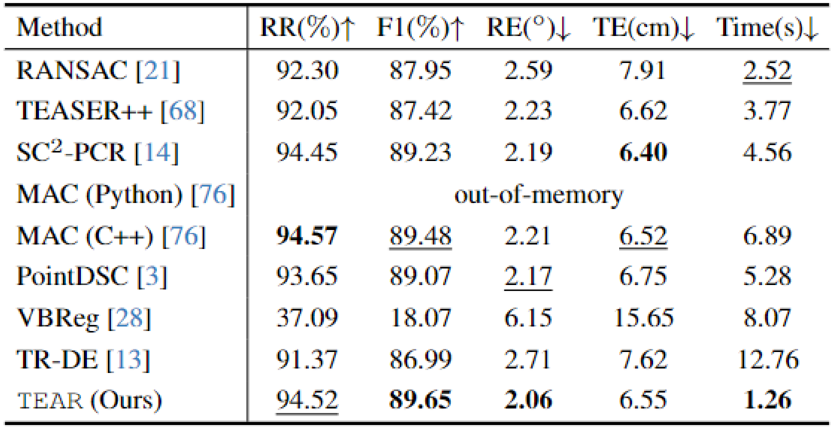




Point Cloud Name	*Armadillo*	*Happy Buddha*	*Asian Dragon*	*Thai Statue*	*Lucy*
# of Input Point Pairs (Outlier Ratio)	10^5 (99%)	5×10^5 (99.2%)	10^6 (99.4%)	4×10^6 (99.6%)	10^7 (99.8%)
Consistency Graph-based	out-of-memory				
Deep Learning-based	out-of-memory				
RANSAC	53.1 | 23.1 | 95.9	30.7 | 15.8 | 582	36.5 | 22.7 | 1179	37.1 | 24.6 | 6125	$\geq$ 8 hours
FGR	57.1 | 39.1 | 2.48	84.1 | 23.7 | 19.3	62.1 | 19.7 | 39.8	79.7 | 15.2 | 175	88.9 | 11.5 | 449
GORE	0.67 | 0.52 | 6592	$\geq$ 12 hours			
TR-DE	36.5 | 16.9 | 4658	$\geq$ 9 hours			
TEAR	0.51 | 0.25 | 12.7	0.23 | 0.13 | 119	0.14 | 0.12 | 356	0.11 | 0.08 | 1013	0.07 | 0.06 | 1972

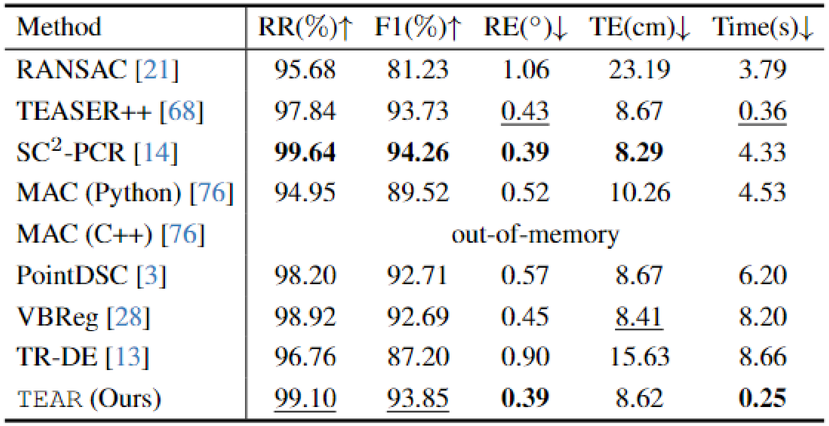
Evaluation Experiments on Real-world Datasets

Table 1. Results on 3DMatch (3DSmoothNet descriptors).



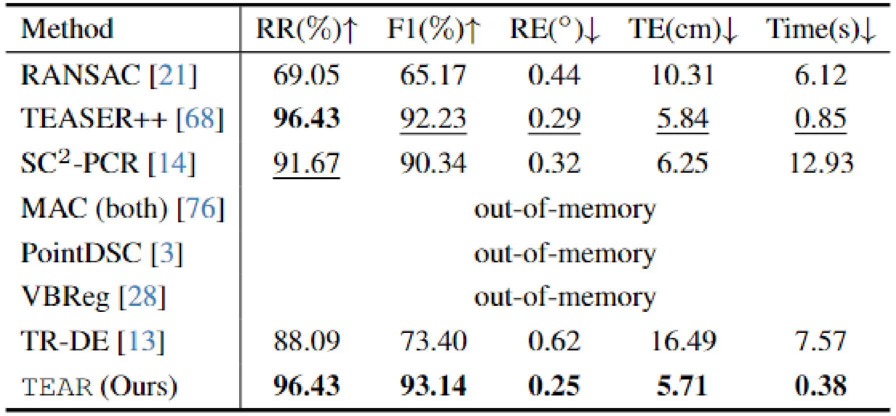
Method	RR(%)↑	F1(%)↑	RE(°)↓	TE(cm)↓	Time(s)↓
RANSAC [21]	92.30	87.95	2.59	7.91	2.52
TEASER++ [68]	92.05	87.42	2.23	6.62	3.77
SC^2-PCR [14]	94.45	89.23	2.19	**6.40**	4.56
MAC (Python) [76]			out-of-memory		
MAC (C++) [76]	**94.57**	89.48	2.21	6.52	6.89
PointDSC [3]	93.65	89.07	2.17	6.75	5.28
VBReg [28]	37.09	18.07	6.15	15.65	8.07
TR-DE [13]	91.37	86.99	2.71	7.62	12.76
TEAR (Ours)	94.52	**89.65**	**2.06**	6.55	**1.26**

Table 2. Results on the KITTI dataset (FPFH descriptors).



Method	RR(%)↑	F1(%)↑	RE(°)↓	TE(cm)↓	Time(s)↓
RANSAC [21]	95.68	81.23	1.06	23.19	3.79
TEASER++ [68]	97.84	93.73	0.43	8.67	0.36
SC^2-PCR [14]	**99.64**	**94.26**	**0.39**	**8.29**	4.33
MAC (Python) [76]	94.95	89.52	0.52	10.26	4.53
MAC (C++) [76]			out-of-memory		
PointDSC [3]	98.20	92.71	0.57	8.67	6.20
VBReg [28]	98.92	92.69	0.45	8.41	8.20
TR-DE [13]	96.76	87.20	0.90	15.63	8.66
TEAR (Ours)	99.10	93.85	**0.39**	8.62	**0.25**

Table 3. Results on the ETH dataset (ISS + FPFH descriptors).



Method	RR(%)↑	F1(%)↑	RE(°)↓	TE(cm)↓	Time(s)↓
RANSAC [21]	69.05	65.17	0.44	10.31	6.12
TEASER++ [68]	**96.43**	92.23	0.29	5.84	0.85
SC^2-PCR [14]	91.67	90.34	0.32	6.25	12.93
MAC (both) [76]			out-of-memory		
PointDSC [3]			out-of-memory		
VBReg [28]			out-of-memory		
TR-DE [13]	88.09	73.40	0.62	16.49	7.57
TEAR (Ours)	**96.43**	**93.14**	**0.25**	**5.71**	**0.38**

Conclusion

- We formulate the outlier-robust 3D registration problem using the robust loss that we call TEAR, a shorthand for Truncated Entry-wise Absolute Residuals.
- We decompose TEAR into two subproblems of dimensions 3 and 2, respectively, which facilitates developing branch-and-bound implementations for globally optimal solutions.
- We derive upper and lower bounds that can be computed via solving a specific 1-dimensional problem in O(NlogN) time, where N is the total number of point pairs.
- Experiments demonstrate that our method can handle more than ten million point pairs with 99.8% random outliers, a setting in which no existing methods have been shown to succeed.